

International conference for the management of risk factors in economically relevant human activities

Viterbo, September 1st-3rd, 2005

Extreme Economic Value-at-Risk (EE-VaR)

Amadeo Alentorn, Sheri Markose

Centre of Computational Finance and Economic Agents
University of Essex, UK



CCFEA

Objectives of the paper

- Introduce a *new risk measure* estimated from information contained in traded option prices.
- Capture the stylized facts of the implied densities extracted from option prices:
 - Left skewness
 - Excess kurtosis (fat tail)
- Introduce a methodology for scaling this risk measure at different time horizons.
- Compare the performance of this new risk measure with the conventional Value at Risk.

Value at Risk

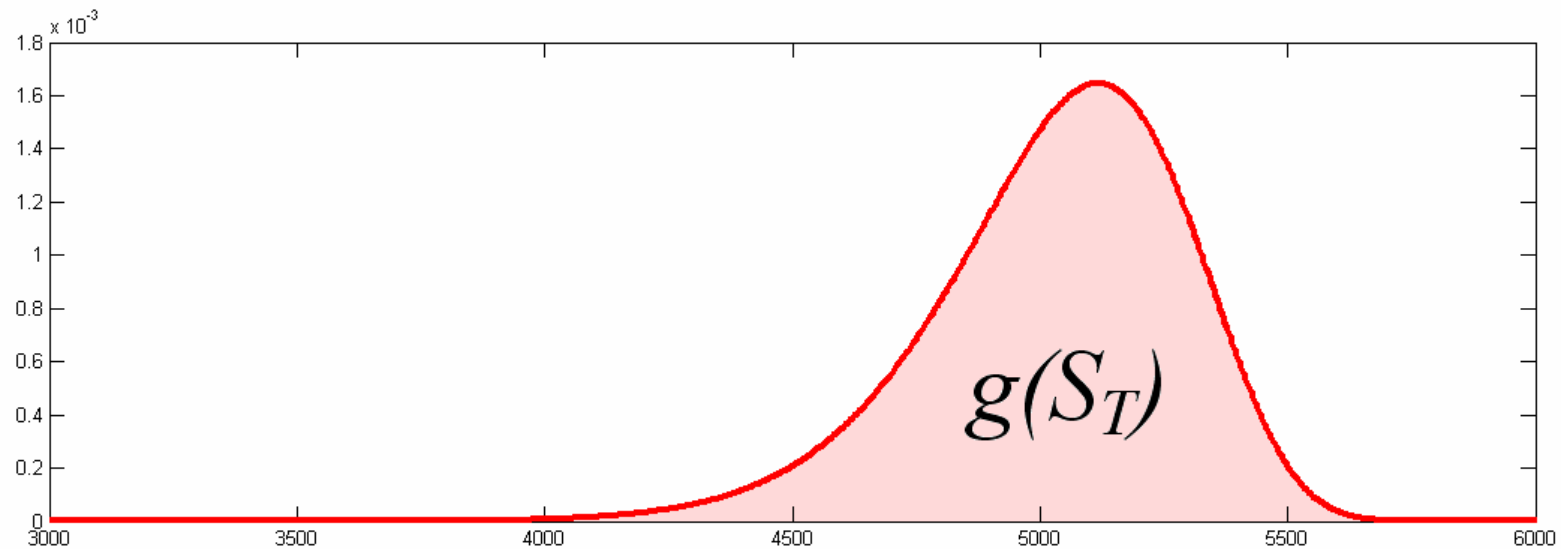
- One of the most popular risk measures in finance.
- It is an estimate, with a given degree of confidence, of how much can be lost from a portfolio over a given time horizon.
- One of the simplest methods to estimate VaR is the Historical method, based in order statistics.
 - The 99% 1-day VaR of a portfolio is the third largest lost in the previous 250 trading days.

Economic VaR (E-VaR)

- Introduced by Ait-Sahalia and Lo (2000), and calculated from an option implied Risk Neutral Density (RND).
- It is a forward looking measure, since the implied RND gives us information about the market expectation of future uncertainty.

The Risk Neutral Density (RND)

- Traded option prices contain information about the market expectations of the future underlying price movements.
- Using a cross-section of option prices, we can extract the Risk Neutral market expectation of the underlying price density at maturity T



Extreme Economic VaR (EE-VaR)

- We propose a new measure of risk: EE-VaR.
- It is also calculated from the implied RND, but the RND is based on the Generalized Extreme Value (GEV) distribution.
- The GEV distribution is very flexible, and is able to model various levels of skewness and kurtosis.

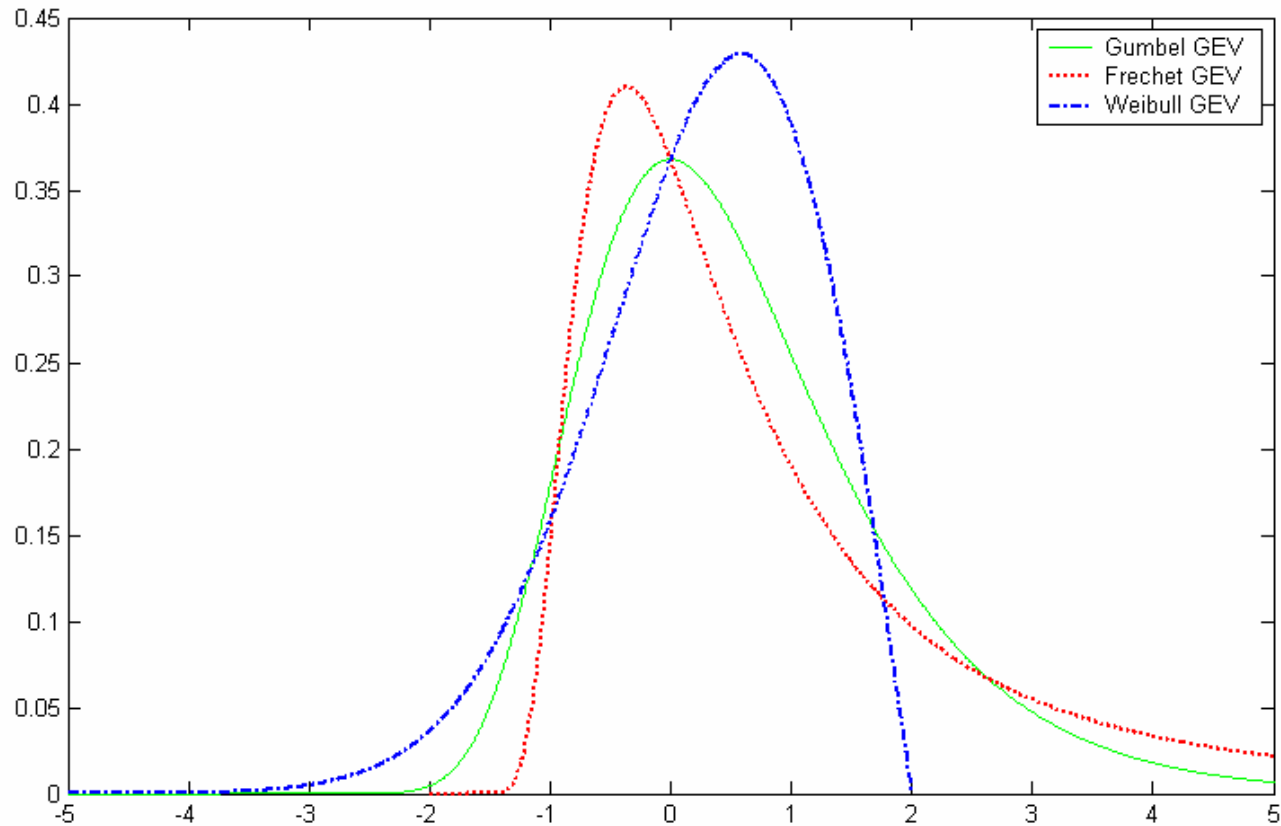
The GEV distribution

- The GEV distribution is given by:

$$G(x) = \begin{cases} \exp\left(-\left(1 + \xi\left(\frac{x - \mu}{\sigma}\right)\right)^{-1/\xi}\right) & \text{for } \xi \neq 0, \text{ with } 1 + \xi\left(\frac{x - \mu}{\sigma}\right) > 0 \\ \exp\left(-\exp\left(-\frac{x - \mu}{\sigma}\right)\right) & \text{for } \xi = 0 \end{cases}$$

- where:
 - μ is the location parameter
 - σ is the scale parameter
 - ξ is the shape parameter

The GEV for different values of ξ



The option pricing model

- From option theory, we know there is a risk neutral density (RND) function, $g(S_T)$, such that the price of a call option can be written as:

$$\begin{aligned} C_t(K) &= E_t^Q \left(e^{-r(T-t)} \max(S_T - K, 0) \right) \\ &= e^{-r(T-t)} \int_K^{\infty} (S_T - K) g(S_T) dS_T \end{aligned}$$

- The expectation $E^Q[\cdot]$ is taken under the risk neutral measure.

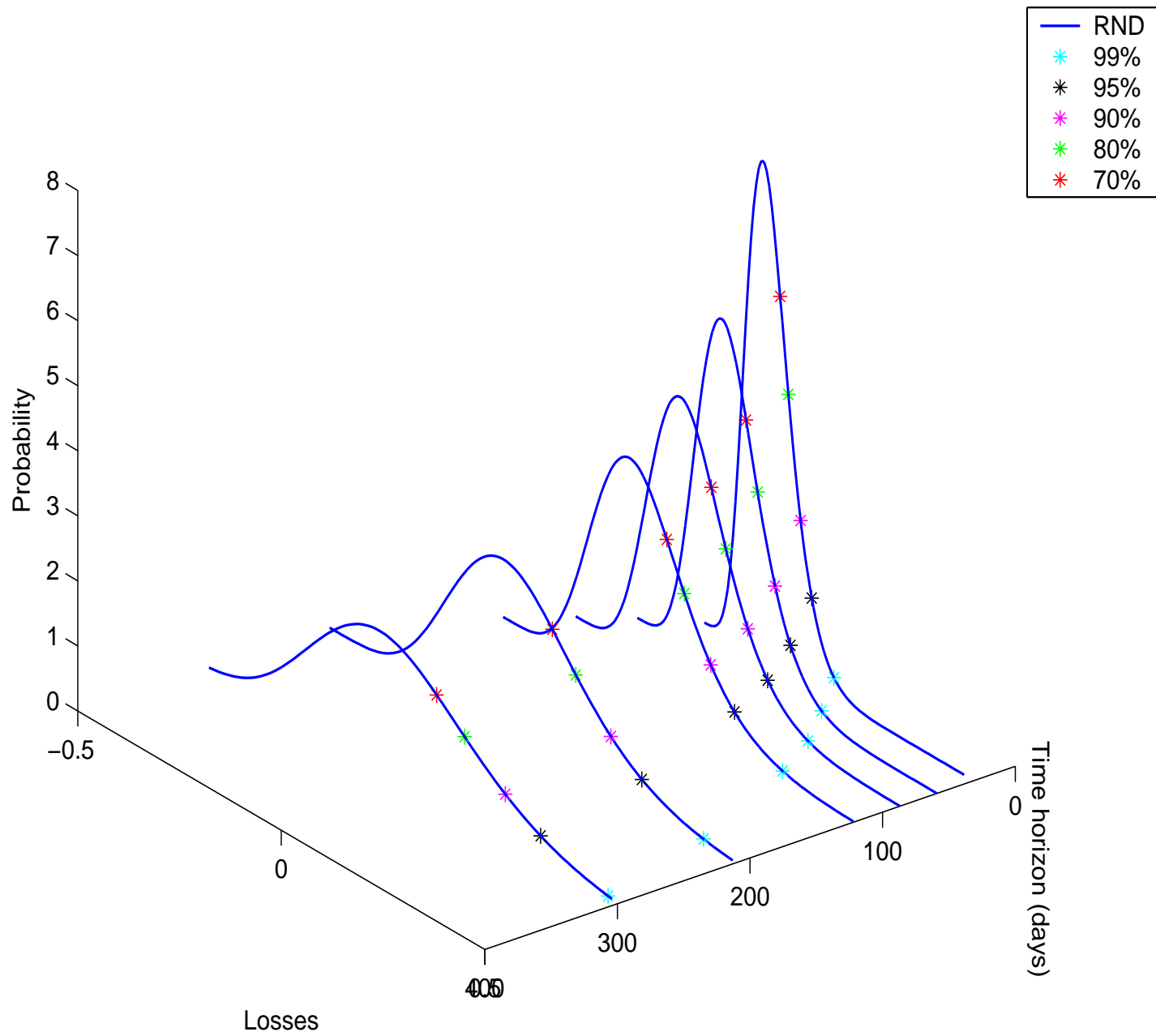
Methodology of RND estimation

- For a given day, we have a set of traded option prices with the same maturity, but different strikes.
- We use a non-linear least squares algorithm to find the set of parameters that minimize the sum of squared errors:

$$SSE(t) = \min_{\zeta, \mu, \sigma} \sum_{i=1}^N \left(C_t(K_i) - \tilde{C}_t(K_i) \right)^2$$

Term structure of RNDs

- However, in most markets, there are option contracts trading for different maturities.
- In the UK, there are options with maturities on the closest 3 months, and also quarterly (Mar, Jun, Sep, and Dec).
- At any given day, we can extract from traded option prices a term structure of RNDs.



EE-VaR calculation

- The EE-VaR values are calculated as the quantile of the GEV density at each confidence interval.

Expiry month	Days to maturity	Number options	EE-VaR				
			70%	80%	90%	95%	99%
Sep-01	31	44	2.4%	4.4%	7.4%	10.1%	15.6%
Oct-01	59	31	3.1%	5.9%	10.2%	14.0%	21.7%
Nov-01	87	13	3.7%	7.2%	12.7%	17.5%	27.4%
Dec-01	122	16	4.2%	8.5%	15.0%	20.8%	32.6%
Mar-02	213	13	5.7%	11.4%	20.1%	27.7%	42.8%
Jun-02	304	10	6.9%	13.7%	23.8%	32.4%	49.0%

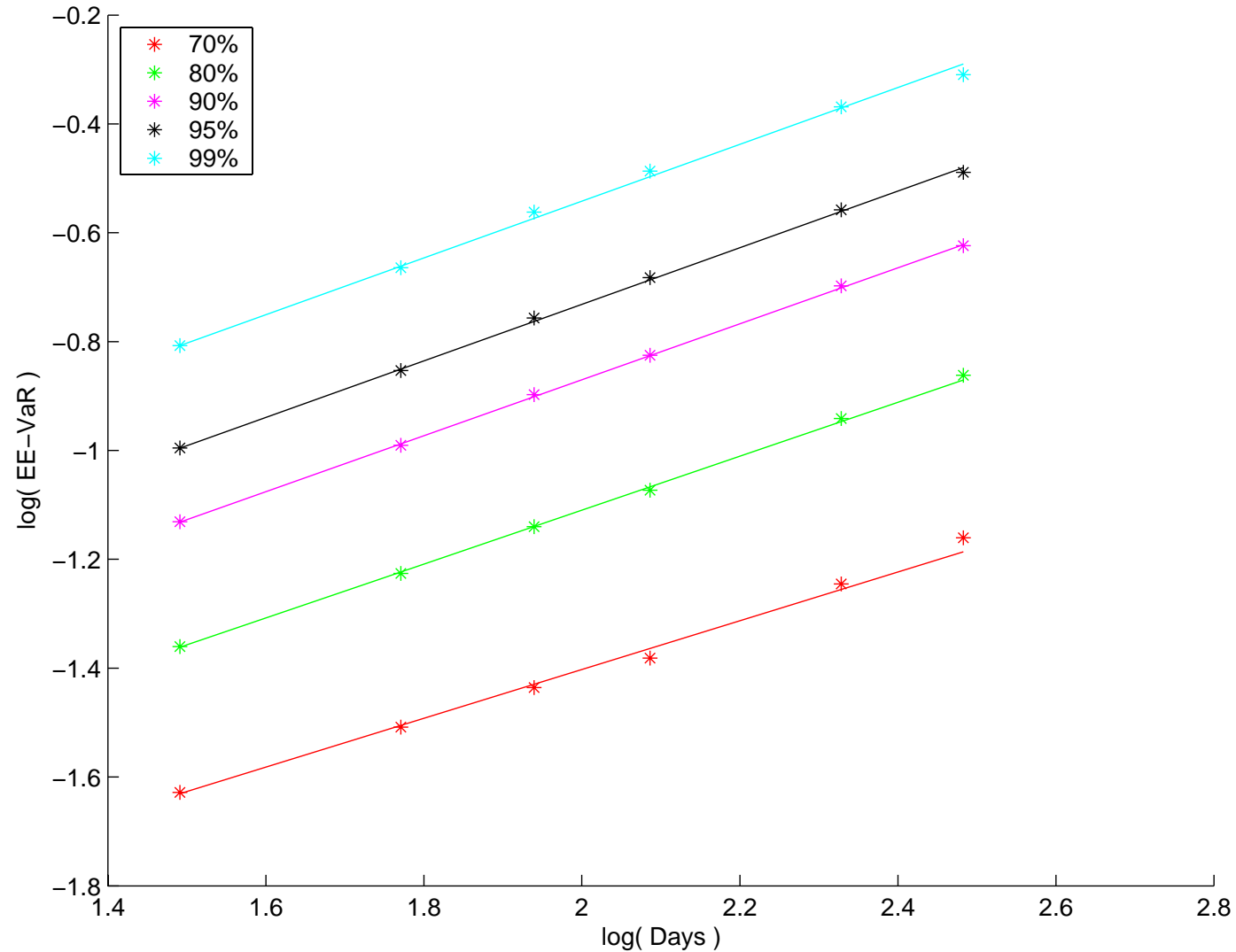
Empirical scaling law

- Financial institutions are usually interested in the 10 day VaR, but the option contracts only expiry once a month.
- Therefore, we need to find a scaling law. The popular \sqrt{T} should not be used, due to the non-Gaussian nature of the data.
- The EE-VaR values appear to be linear with days to maturity k , in a log-log scale.

$$\log(EEVaR(k, cl)) = b \log(k) + c$$

- b : the slope
- c : the intercept, equal to the log of the 1 day EE-Var

Regression in quantile space



Data description

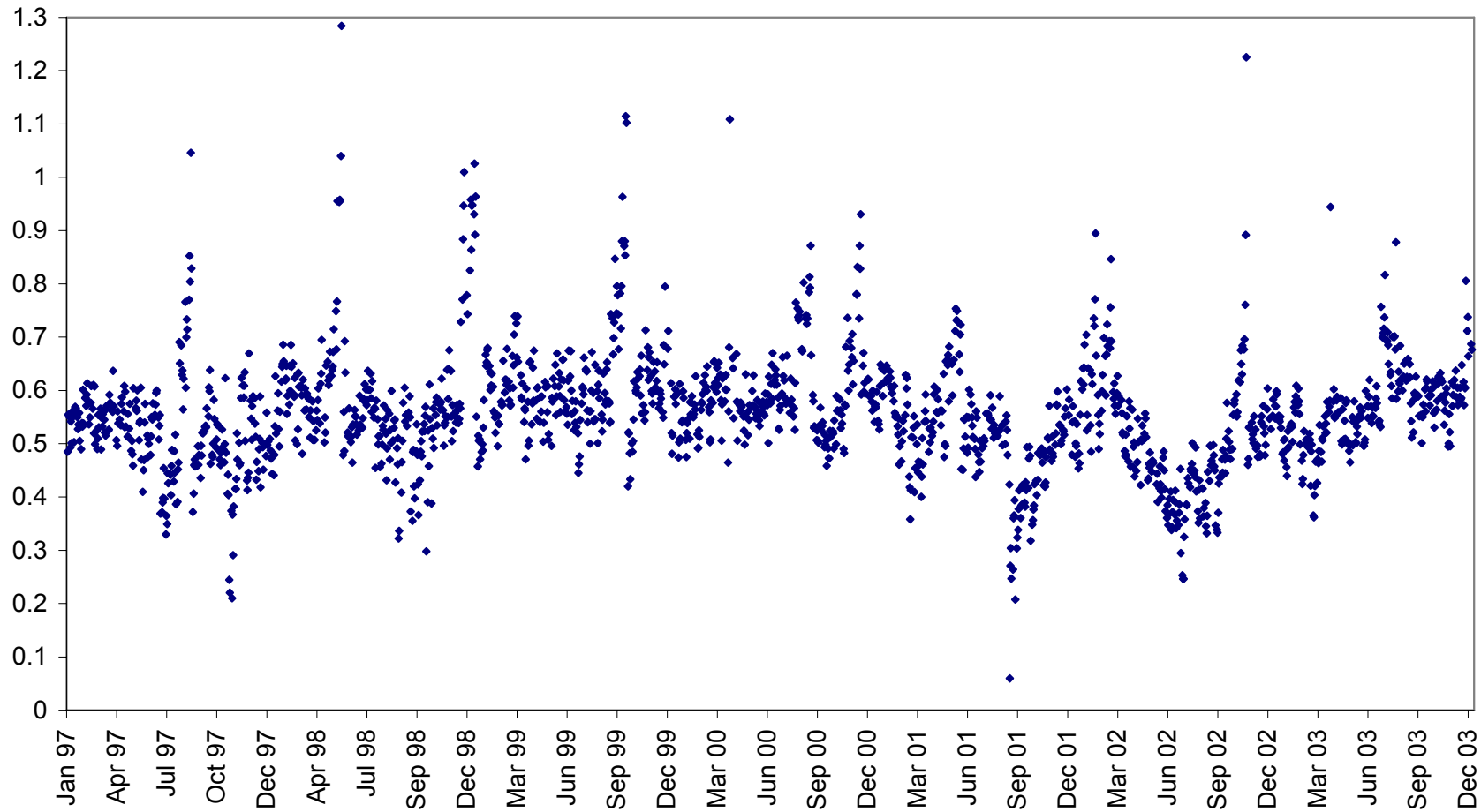
- We used closing prices of traded European options on the FTSE 100.
- Period of study 1997 – 2003
- Interest rate: 3-months LIBOR rate.

Regression results

- The average slope and intercept for each quantile, calculated for each day between 1997-2003.

Confidence level	<i>b</i>	<i>c</i>	1-day EE-VaR = 10^c
70%	0.41 (0.12)	-2.24 (0.26)	0.6%
80%	0.48 (0.09)	-2.04 (0.23)	0.9%
90%	0.51 (0.09)	-1.84 (0.23)	1.4%
95%	0.53 (0.09)	-1.73 (0.23)	1.9%
99%	0.56 (0.11)	-1.57 (0.25)	2.7%
Average	0.50	-1.88	1.5%

Time varying scaling law



Time series of the b estimates (the slope) at the 99% confidence level.

Backtesting performance

- The performance of a VaR methodology is assessed by calculating how close the percentage violations of the predicted VaR value is to the benchmark.
 - For example, at the 99% confidence level, we should exceed the predicted VaR value 1% of the time. If $> 1\%$, we are underestimating risk.

Backtesting results

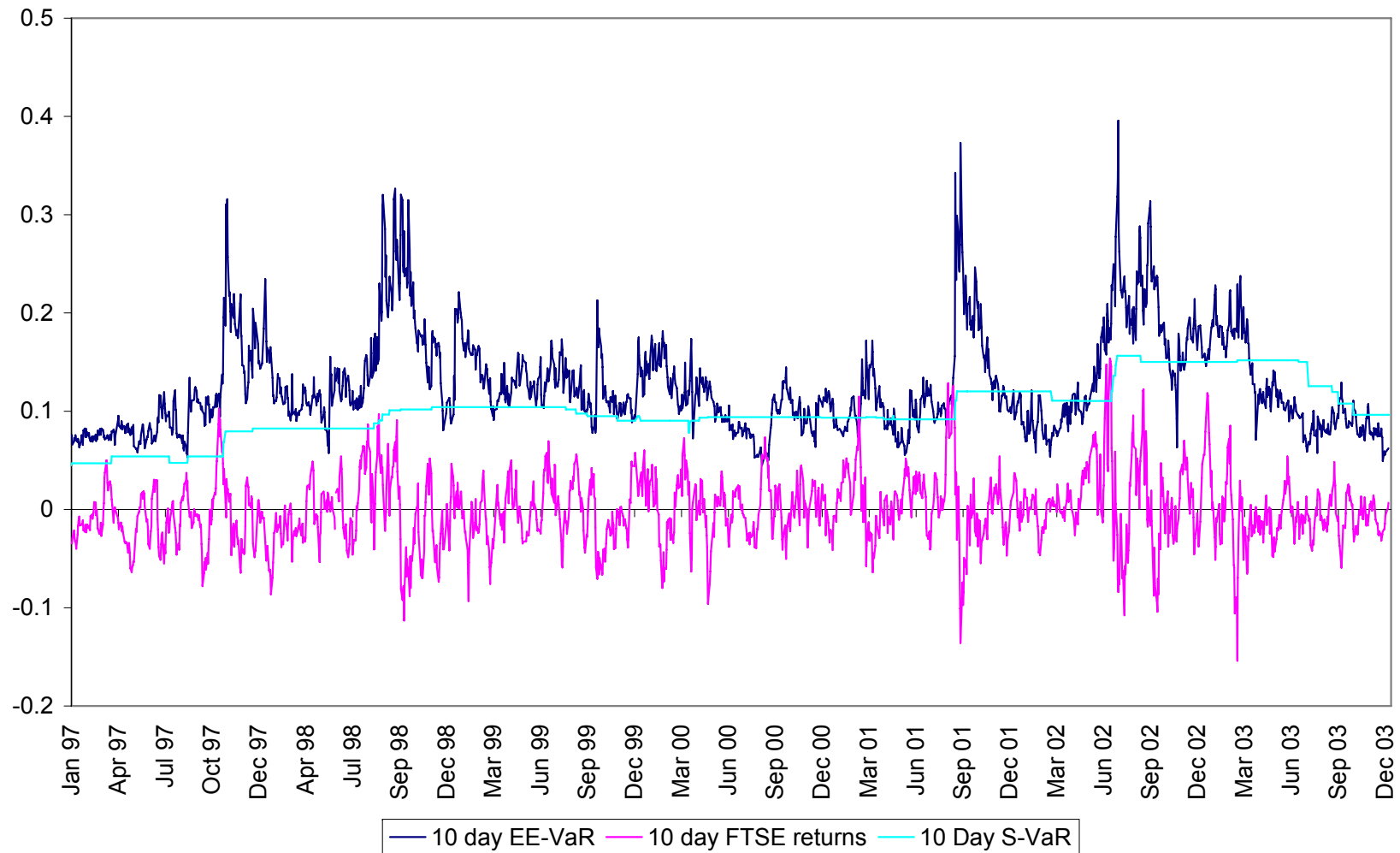
Horizon (days)	Confidence level				
	95	96	97	98	99
Benchmark	5	4	3	2	1
1	6.2	5.5	4.4	3.8	3.1
10	2.4	1.8	1.5	1.3	0.6
30	2.8	2.0	1.3	0.5	0.1
60	2.5	2.2	1.3	0.7	0.0
90	2.9	2.3	1.5	0.8	0.0
Average	3.4	2.8	2.0	1.4	0.8

Percentage violations of EE-VaR

Horizon (days)	Confidence level				
	95	96	97	98	99
Benchmark	5	4	3	2	1
1	5.9	4.7	3.8	2.5	1.5
10	4.6	4.0	3.1	1.7	1.4
30	5.3	4.2	3.3	2.1	0.8
60	4.3	3.6	2.8	1.1	0.3
90	4.5	2.8	1.9	0.7	0.0
Average	4.9	3.9%	3.0	1.6	0.8

Percentage violations of (statistical) VaR (S-VaR)

EE-VaR vs. S-VaR



Capital requirements

- The VaR estimates are used to impose a bank's capital requirement.
- The average capital requirement based on the EE-VaR and S-VaR for the 10 day VaR at different confidence levels

	Confidence level					Average
	95%	96%	97%	98%	99%	
S-VaR	6.5% (1.9%)	7.3% (2.3%)	7.8% (2.4%)	9.1% (2.9%)	10.1% (2.8%)	8.2% (2.5%)
EE-VaR	8.1% (3.1%)	8.7% (3.3%)	9.6% (3.7%)	10.8% (4.2%)	12.7% (5.0%)	9.9% (3.9%)

Conclusions

- There is a clear tradeoff between the frequency of VaR violations, and the capital requirements.
- EE-VaR gives fewer cases of benchmark violations, but yields higher and more volatile capital requirements than S-VaR.
- Regulators may prefer EE-VaR since it is more conservative, but financial institutions are more concerned with savings on capital requirements.
- We suggest the use of EE-VaR as an additional tool in Risk Management, but not as a replacement of S-VaR, since it contains different information.

Future work

- We have used unadjusted RND. Some studies conclude that the forecasting performance of RNDs improves when adjusting them by the risk premium.
- Test the robustness of the EE-VaR estimates, given that the tails of the distribution are estimated with very few, if any, traded option prices.